33.12 An energy storage facility uses pumped hydro to capitalize on the low cost of off-peak electricity of $0.02/kWh. 400 GPM of 60°F water is pumped from a low reservoir to a high reservoir at 1000 ft higher elevation through an equivalent length of 3000 ft of 5 inch schedule 40 steel pipe using a pump driven by an AC motor. The overall efficiency of the pump set is 75%. Assuming the off-peak operation lasts for 8 hours per night, what is the cost of electricity for one night?

A. $2

B. $13

C. $17

D. $52

The cost depends on the run time, the unit rate for electricity, and the electrical power demand, the latter which is unknown. The electrical power, P, depends on the hydraulic horsepower, whp, which can be determined using the volume flow rate, head added by the pump, and the efficiency. Note the efficiency given is the *total* efficiency including the pump losses and motor losses, yielding electrical power directly from hydraulic horsepower, skipping the intermediate step of specifying the brake horsepower, which is not a required step.

$$P = \frac{whp}{\eta} = \frac{Q_{[gpm]}\Delta h_{[ft]}}{3960\eta}$$

Write the form of the Bernoulli Equation suited for pumping applications.

$$\Delta h = h_A = \frac{P_2 - P_1}{\gamma} + \frac{v_2^2 - v_1^2}{2g} + z_2 - z_1 + h_f$$

Although the two reservoirs are at different elevations, presumably both are exposed to atmospheric pressure, and the column of air associated with the elevation difference does not have a meaningful impact on the static pressure. Therefore:

$$P_1 \approx P_2 \rightarrow \frac{P_2 - P_1}{\gamma} \approx 0$$

Since the volume of the reservoirs is large compared to the entering flow, the velocity term is also negligible.

$$v_1 \approx v_2 \approx 0$$

$$\frac{v_2^2 - v_1^2}{2g} \approx 0$$

The difference in height is given.

$$\Delta z = z_2 - z_1 = 1000 ft$$

To find the losses, write the Darcy Equation.

$$h_f = \frac{fLv^2}{2gD}$$

Using the Steel Pipe Friction Tables for $400gpm$ flowing through a nominal $5inch$ standard weight steel pipe.

$$D = 5.047in\left(\frac{1ft}{12in}\right) = 0.4206ft$$

$$Q = 400gpm$$

$$v = 6.41\frac{ft}{s}$$

Find the relative roughness:

$$\frac{\epsilon}{D} = \frac{.0002ft}{.4206ft} \approx 0.0005$$

Find the Reynolds Number. Use the Properties of Water table to look up the Kinematic Viscosity.

$$Re = \frac{vD}{\nu} = \frac{\left(6.41\frac{ft}{s}\right)(0.4206ft)}{1.217\times10^{-5}\frac{ft^2}{s}} = 220,000 \approx 2\times10^5$$

Use the Moody Diagram to find the friction factor.

$$f = f(Re, \frac{\epsilon}{D}) = 0.019$$

Substitute and solve the Darcy Equation.

$$h_f = \frac{fLv^2}{2gD} = \frac{(.019)(3000ft)\left(6.41\frac{ft}{s}\right)^2}{2(.4206ft)\left(32.2\frac{ft}{s^2}\right)} = 86.5ft$$

Determine the head added by the pump.

$$h_A = 1000ft + 86.5ft = 1086.5ft$$

Determine the electrical power demand and convert to KW.

$$P = \frac{whp}{\eta} = \frac{Q_{[gpm]}\Delta h_{[ft]}}{3960\eta} = \frac{(400)(1086.5)}{3960(0.75)} = 146.3hp\left(\frac{0.7457KW}{1hp}\right) = 109.1KW$$

Calculate the cost for one night of off-peak operation:

$$Cost = (109.1KW)(8hr)\left(\frac{\$.02}{KWH}\right) = \$17.46$$

Answer C